**Physics 200B** **Mechanics II** **Winter 2013**

**Problem Set V**: Optional. The purpose of this set is to help you prepare for the Final Exam.

1.) In ideal MHD, the magnetic field evolves according to Ohm's Law $\underline{E}+\underline{v}\times\underline{B}/c=0$, and Faraday's Law. Here $\underline{v}=\underline{v}(\underline{x},t)$ is the fluid velocity. Thus, field and fluid form a coupled system in MHD.

a.) Prove that $\underline{B}(\underline{x},t)/\rho(\underline{x},t)$ is 'frozen in' to the fluid in compressible MHD.

b.) Prove the magnetic analogue of Kelvin's Theorem, namely that

$$\frac{d}{dt}\int_A \underline{B}\cdot da = 0$$

i.e. the magnetic flux thru a closed loop moving with the fluid is conserved.

In both cases, use analogy with what was done for vorticity.

2a.) A spherical mass $m$ of radius $R$ is attached to a spring $k$ and immersed in an infinite ideal fluid. Assuming the flow is potential flow, what is the frequency of the motion?

b.) Now consider the analogous problem for a pendulum of length $\ell$.

3.) Consider a sphere of radius $R$, which pulsates about $R_0$, i.e. $R=R_0+R_1e^{-iwt}$ with $R_1/R_0 \ll 1$ in a fluid. Assume $c_s/w > R$.

a.) Calculate the time-averaged acoustic energy distribution in the far field.

b.) What is the force felt by an object in the far field, with surface area $A$ facing the pulsing sphere?

4.) Consider a wave train propagating along an infinite string with tension $T$ and $\mu=\mu(x)$, with $d\mu/dx<0$. Assume all spatial variation is slow.

a.) Derive an equation for the evolution of the wave action density.

b.) Use your result for a.) to determine how the wave amplitude varies with position.

5.) Consider a sphere moving at constant velocity $\underline{u}$ a distance *d* below a free surface. The sphere moves parallel to the surface.

a.) For a free surface, ignoring coupling to surface waves, etc., the boundary condition is $P = 0$. Calculate the flow pattern induced by the sphere.

b.) What might happen if surface wave coupling were taken into account? Be as quantitative as possible.

6.) Consider an elastic sphere which is initially compressed and then released in a fluid. Assuming the sphere is perfectly elastic, calculate the rate of decay of pulsations due to acoustic radiation.